USN | | | | | | | | | | | **10MAT21**

Second Semester B.E. Degree Examination, June 2012

Engineering Mathematics - II

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer FIVE full questions choosing at least two from each part.***
2. ***Answer all objective type questions only on OMR sheet page 5 of the answer booklet.***
3. ***Answer to objective type questions on sheets other than OMR will not be valued.***

PART – A

1. a. Select the correct answer : **(04 Marks)**

i) We say that the given differential equation is solvable for x, if it is possible to express x interms of

A) x and y B) x and p C) y and p D) x, y and p

ii) The general solution of $P^2 - 7P + 12 = 0$ is

A) $(y + 3x - c)(y + 4x - c) = 0$ B) $(y - 3x - c)(y - 4x - c) = 0$
C) $(y - 4x)(y + 3x) = 0$ D) None of these

iii) The general solution of the equation $y = 3x + \log P$ is

A) $y = 3x + 3 + c\,e^y$ B) $y = 3x + \log(3 + c\,e^y)$
C) $y + 3x = 3 + c\,e^y$ D) None of these

iv) The general solution of the equation $(y - Px)^2 = 4P^2 + 9$ is

A) $y = c\,x + \sqrt{4c^2+9}$ B) $y = c + \sqrt{4c^2+9}$
C) $y = c\,x + \sqrt{4c^2-9}$ D) $y - c\,x = 4\,c^2 + 9$

b. Solve : $p^2 + 2py \cot x = y^2$. **(05 Marks)**

c. Solve : $p^2 + 4\,x^5 p - 12x^4\,y = 0$, obtain the singular solution also. **(05 Marks)**

d. Solve the equation $(px - y)(py + x) = 2p$ by reducing into Clairaut's form, taking the substitution $X = x^2$, $Y = y^2$. **(06 Marks)**

2. a. Select the correct answer : **(04 Marks)**

i) P.I. of $y'' - 3y' + 2y = 12$ is

A) 6 B) $y = c_1 e^x + c_2\ e^{2x}$ C) $\frac{1}{12}$ D) $\frac{1}{6}$

ii) The complementary function of $(D^4 - a^4)\,y = 0$ is

A) $y = c_1 e^{ax} + c_2 e^{-ax} + c_3 \cos x + c_4 \sin x$
B) $y = c_1 e^{-ax} + c_2 e^{ax}$
C) $y = c_1 e^{ax} + c_2 e^{-ax} + c_3 \cos ax + c_4 \sin ax$
D) None of these

iii) If $F(D) = D^2 + 5$, $\frac{1}{f(D)} \sin 2x = \ldots\ldots.$

A) $\frac{-\cos 2x}{2}$ B) $\frac{\cos 2x}{2}$ C) $\sin 2x$ D) $\cos 2x$

iv) The solution of the differential equation $y'' - 3y' + 2y = e^{3x}$ is

A) $y = c_1 e^{-x} + c_2 e^{-2x} + \frac{1}{2} e^{3x}$ B) $y = c_1 e^{x} + c_2 e^{2x} + \frac{1}{2} e^{3x}$
C) $y = c_1 e^{-x} + c_2 e^{2x} + \frac{1}{2} e^{3x}$ D) $y = c_1 e^{-x} + c_2 e^{2x} + \frac{1}{2} e^{-3x}$

b. Solve : $(D - 2)^2 y = 8 (e^{2x} + \sin 2x)$. **(05 Marks)**

c. Solve : $y'' - 2y' + y = x \cos x$. **(05 Marks)**

d. Solve $\frac{dx}{dt} - 2y = \cos 2t$, $\frac{dy}{dt} + 2x = \sin 2t$, given that x = 1, y = 0 at t = 0. **(06 Marks)**

3. a. Select the correct answer : **(04 Marks)**

i) The Wronskian of x and e^x is

A) $e^x (1-x)$ B) $x e^x$ C) $e^{-x} (x-1)$ D) $e^x (x-1)$

ii) In the equation $\frac{dx}{dt} + y = \sin t + 1$, $\frac{dy}{dt} + x = \cos t$, if $y = \sin t + 1 + e^{-t}$, then x = ….

A) 0 B) e^{-t} C) $x e^{-t}$ D) e^t

iii) In homogeneous linear differential equation whose auxiliary equation has roots 1, -1 is

A) $y'' + y = 0$ B) $x^2 y'' - xy' - y = 0$

C) $x^2 y'' + xy' - y = 0$ D) $y'' - y' = 0$

iv) The solution of $x^2 y'' + xy' = 0$ is

A) $y = c_1 + c_2 \log x$ B) $y = a \log x + 6$ C) $y = e^t$ D) $y = e^{-t}$

b. Using the method of variation of parameters solve $y'' + 4y = \tan 2x$. **(05 Marks)**

c. Solve : $(1+x)^2 y'' + (1+x) y' + y = 2 \sin [\log (1 + x)]$. **(05 Marks)**

d. Solve by Frobenius method, the equation

$$4x \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0.$$ **(06 Marks)**

4. a. Select the correct answer : **(04 Marks)**

i) The solution of $\frac{\partial^2 z}{\partial y^2} = \sin (xy)$ is

A) $z = -x^2 \sin(xy) + y f(x) + \phi (x)$ B) $z = \frac{\cos(xy)}{x^2} + y f(x) + \phi (x)$

C) $z = -\frac{\sin(xy)}{x^2} + y f(x) + \phi (x)$ D) None of these

ii) A solution of $(y-z) p + (z-x) q = x - y$ is

A) $x^2 + y^2 + z^2 = f (x + y + z)$ B) $x^2 - y^2 - z^2 = f (x - y + z)$

C) $x^2 - y^2 - z^2 = f (x - y - z)$ D) None of these

iii) The partial differential equation obtained form z = ax + by + ab by eliminating a and b is

A) z = px + qy B) z = px + qy + pq

C) z = px + qy - pq D) z = px – qy - pq

iv) The partial differential equation obtained from z = f(x + y) + g(x – y) by eliminating the arbitrary functions is

A) r + t = 0 B) r – t = 0 C) $r - a^2 t = 0$ D) $r + a^2 t = 0$

b. Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$, given that when x = 0, $z = e^y$ and $\frac{\partial z}{\partial x} = 1$. **(05 Marks)**

c. Solve : $(x^2 - yz) p + (y^2 - zx) q = z^2 - xy$. **(05 Marks)**

d. Solve by the method of variables $4 \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$, given that $u(0, y) = 2 e^{5y}$.

(06 Marks)

PART - B

5. a. Select the correct answer : **(04 Marks)**

i) The value of $\int_1^2 \int_1^3 x\,y^2\,dx\,dy$ is ______

A) 0 B) 1 C) $\frac{13}{2}$ D) 13

ii) The integral $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy\ dx$ after changing the order of integration is

A) $\int_0^\infty \int_0^y \frac{e^{-y}}{y} dx\,dy$ B) $\int_0^\infty \int_y^\infty \frac{e^{-y}}{y} dx\ dy$

C) $\int_0^\infty \int_0^\infty \frac{e^{-y}}{y} dx\ dy$ D) $\int_0^\infty \int_0^x \frac{e^{-y}}{y} dx\ dy$

iii) $B\left(\frac{1}{2},\frac{1}{2}\right) = \ldots..$

A) $\sqrt{\pi}$ B) $\frac{\sqrt{\pi}}{2}$ C) 3.1416 D) $-\pi$

iv) In terms of Beta function $\int_0^{\pi/2} \sin^7\theta\sqrt{\cos\theta}\ \ d\theta = \ldots\ldots$

A) $\beta\,(4, \frac{3}{4})$ B) $\frac{1}{2}\beta\,(4, \frac{3}{4})$ C) $\beta\,(2, \frac{3}{2})$ D) $\frac{1}{2}\beta\,(2, \frac{3}{2})$

b. Change the order of integration in $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2\,dx\,dy$ and hence evaluate the same. **(05 Marks)**

c. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\,dz\,dy\,dx.$ **(05 Marks)**

d. Show that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}}\ x \int_0^{\pi/2} \sqrt{\sin\theta}\ \ d\theta = \pi.$ **(06 Marks)**

6 a. Select the correct answer : **(04 Marks)**

i) In Green's theorem in the plane $\oint_C m\ dx + n\ dy = \ldots\ldots$

A) $\iint_R \left(\frac{\partial m}{\partial y} + \frac{\partial n}{\partial x}\right) dx\ dy$ B) $\iint_R \left(\frac{\partial m}{\partial y} - \frac{\partial n}{\partial x}\right) dx\ dy$

C) $\iint_R \left(\frac{\partial n}{\partial x} - \frac{\partial m}{\partial y}\right) dx\ dy$ C) $\iint_S \vec{F}.\ \hat{n}\ \ ds$

ii) The area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by employing Green's theorem is

A) 0 B) 1 C) π D) π a b

iii) A necessary and sufficient condition that the line integral $\int_L \vec{F}.\ dR$ for every closed curve C is

A) curl $\vec{F} = 0$ B) div $\vec{F} = 0$ C) curl $\vec{F} \neq 0$ D) div $\vec{F} \neq 0$

iv) If V is the volume bounded by a surface S and $\vec{F}$ is continuously differentiable vector function then $\iiint_V \text{div}\,\vec{F}\ dv = \ldots\ldots$

A) $\oint_e \vec{F}.d\vec{r}$ B) $\iint_S \vec{F}.\ \hat{n}\ ds$ C) $\iint_S (\nabla \times F).\hat{n}\ ds$ D) None of these

b. If $\vec{F} = 2xy\,i + yz^2\,j + xz\,k$ and s is the rectangular parallelepiped bounded by x = 0, y = 0, z = 0, x = 2, y = 1, z = 3, evaluate $\iint_S \vec{F}.\ \hat{n}\ ds$. **(05 Marks)**

c. Using Green's theorem, evaluate $\int_c [(y - \sin x)dx + \cos x\ dy]$, where C is the plane triangle enclosed by the lines y = 0, x = $\frac{\pi}{2}$ and y = $\frac{2x}{\pi}$. **(05 Marks)**

d. Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)i - 2xy\,j$ taken around the rectangle bounded by the lines x = ± a, y = 0 , y = b. **(06 Marks)**

7. a. Select the correct answer : **(04 Marks)**

i) $L\{e^{2(t-1)}\} = \ldots.$

A) $\frac{1}{s-2}$ B) $\frac{e^{-2}}{s-2}$ C) $\frac{1}{s+2}$ D) $\frac{e^{-2}}{s+2}$

ii) $L\{t^{1/2}\} = \ldots\ldots.$

A) $\frac{\sqrt{\pi}}{s^{3/2}}$ B) $\frac{2\sqrt{\pi}}{s^{3/2}}$ C) $\frac{\sqrt{\pi}}{2\sqrt{s}}$ D) $\frac{\sqrt{\pi}}{2s^{3/2}}$

iii) $L\left\{\frac{\sin t}{t}\right\} = \ldots\ldots$

A) $\frac{\pi}{2} + \tan^{-1} s$ B) $\frac{\pi}{2} - \cot^{-1} s$ C) $\cot^{-1} s$ D) $\tan^{-1} s$

iv) $L\{\delta(t+2)\} = \ldots\ldots$

A) e^{-as} B) e^{2s} C) e^{-2s} D) e^{as}

b. Find the value of $\int_0^\infty t^3 e^{-t} \sin t\ dt$ using Laplace transforms. **(05 Marks)**

c. Draw the graph of the periodic function

$$f(t) = \begin{cases} t, & 0 < t < \pi \\ \pi - t, & \pi < t < 2\pi \end{cases}$$ and find its Laplace transform. **(05 Marks)**

d. Prove that $L\{\delta\{t-a)\} = e^{-as}$. **(06 Marks)**

8. a. Select the correct answer : **(04 Marks)**

i) $L^{-1}\left\{\frac{1}{4s^2 - 36}\right\} = \ldots\ldots\ldots$

A) $\frac{1}{4}\cos h\ 6t$ B) $\frac{1}{12}\sin 3t$ C) $\frac{1}{6}\cos h\ 3t$ D) $\frac{1}{12}\sin h\ 3\,t$

ii) $L^{-1}\left\{\frac{1+e^{-3s}}{s^2}\right\} = \ldots\ldots.$

A) t + (t -3) u (t – 3) B) (t – 3) u (t – 3)

C) t – (t – 3) u (t – 3) D) t + (t + 3) u (t + 3)

iii) $L^{-1}\left\{\cot^{-1}\frac{s}{a}\right\} = \ldots\ldots..$

A) $\frac{\sin t}{t}$ B) $\frac{\sin a t}{t}$ C) $\frac{\sin h a t}{t}$ D) $\frac{\sinh t}{t}$

iv) $L\left[\int_0^t f(u)\, g\,(t-u)\, du\right] = \ldots\ldots\ldots$

A) f(t) g(t) B) f(s) g(s) C) f(s) – g(s) D) $\frac{f(s)}{g(s)}$

b. Find $L^{-1}\left\{\frac{5s+3}{(s-1)(s^2+2s+5)}\right\}$. **(05 Marks)**

c. Apply convolution theorem to evaluate

$L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$. **(05 Marks)**

d. Solve $(D^3 - 3D^2 + 3D - 1)\, y = t^2 e^t$, $y(0) = 1$, $y'(0) = 0$, $y''(0) = -2$ by Laplace transform method. **(06 Marks)**

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